1. (20 points) - In this question, you will solve a boy or girl paradox problem. Martin Gardner prepared this question in 1959 and published in Scientific American.

   a) Mr. Jones has two children. Given that **the older child is a girl**, what is the probability that both children are girls?

Old Young
Girl * Girl GG
Boy GB

$P = \frac{E}{S} = \frac{1}{2}$

1/2

   b) Mr. Smith has two children. Given that **at least one of them is a boy**, What is the probability that both children are boys?

Boy - Girl
Girl - Boy
* Boy - Boy

$P = \frac{E}{S} = \frac{1}{3}$

1/3

2. (20 points) In this question, you will analyze the probability events in a dice-game. Suppose you have 4 dices each with 6 side,

   a) How many different possible outcomes are possible if you roll 4 dices?

Each die . 6 outcomes {1,2,3,4,5,6}

$6^4 = 6 \times 6 \times 6 \times 6$

6^4

   b) Given that all 4 numbers are expected to be different, how many different outcomes are possible?

$6 \times 5 \times 4 \times 3 = 360$

360

   c) What is the probability of getting <u>at least</u> 2 numbers same if you roll 4 dices?

P(at least 2 are) = 1 − P(all numbers different)

$= 1 - \frac{360}{6^4}$

0,72

d) What is the probability of getting all different numbers if we roll 4 dices?

$$P = \frac{\text{Event space}}{\text{Sample space}} = \frac{360}{64}$$

360/64

3. (20 points) Given that there are AATGAGCTTC nucleotides in a short oligo, how many different 10-base pair sets can be generated?

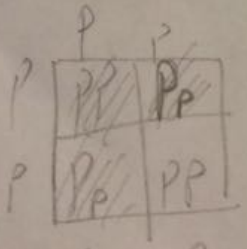
Permutation of indistinct objects

3 A
2 G
3 T
2 C

$$\frac{10!}{3!\,2!\,3!\,2!} = 25200$$

25200

4. (20 points) Parents are heterozygous for a genetic disorder and have 4 children. What is the probability that at least one child will have this genetic disease?



P(D) = 3/4
P(R) = 1/4

1 2 3 4 (children)

$3/4 \times 3/4 \times 3/4 \times 3/4 = \frac{81}{256}$ Unwell Children

0.68

$P(X \geq 1) = 1 - P(X = 0)$

$= 1 - (3/4)^4 = 1 - \frac{81}{256} = 0,68$

5. (10 points) A student tosses a fair 5-sided die to guess the answers on a multiple-choice exam. There are total of 8 questions and each question has 5 possible choices like (A,B,C,D or E).

a) What are the expected average true answers and standard deviation of the number of correct answers?

Average $= n \times p =$ total questions $\times$ probability

$= 8 \times 0,2 = 1.6$

$p = 1/5 = 0,2$

$SD = (n p(1-p))^{1/2} = (8 \times 0,2 \times 0,8)^{1/2} = 1,13$

$\mu = 1,6$
$\sigma = 1,13$

b) What is the probability that a student will get exactly 2 questions correct?

$P(X=2) = \binom{8}{2} (0,2)^2 (0,8)^{8-2}$

$= 28 \times 0,04 \times 0.26$

$= 0,29$

0,29

c) What is the probability that a student will get more than 4 questions correct?

$P(X>4) = 1 - P(X \leq 4) = 1 - 0,97 = 0,03$

$P(X=0) = \binom{8}{0} (0,2)^0 (0,8)^8 = 0,16$

$P(X=1) = \binom{8}{1} (0,2)^1 (0,8)^7 = 0,33$

$P(X=2) = 0,29$ (from part b)

$P(X=3) = \binom{8}{3} (0,2)^3 (0,8)^5 = 0,14$

$P(X=4) = \binom{8}{4} (0,2)^4 (0,8)^4 = 0,05$

0,03

d) What is the probability that a student will get between 3 and 7 questions correct?

$P(3<X<7) \Rightarrow P(X<7) - P(X<3)$

$\Rightarrow P(X=6) + P(X=5) + P(X=4) + P(X=4)$

$\Rightarrow 0.001 + 0.009 + 0.05 = 0.06$

0,06

e) If the passing score for the exam is %50, what is the probability that he will pass the exam?

$8 \times 0{,}50 = 4$ question correct

$P(X \geq 4) = 1 - P(X<4) = 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$

$P(X=0) = 0.16$
$P(X=1) = 0.33$
$P(X=2) = 0.29$
$P(X=3) = 0.14$

$= 1 - 092$
$= 0,08$

0.08

6. (10 points) Given that three is a parent carrying three hybrid (trihybrid) heterozygous genes.

Parent 1 is XxYyZz and Parent 2 XxYyZz.

a) What is the probability that offspring carries 1 dominant and 2 recessive genes?

Xx Xx → XX, Xx, Xx, xx

	X	x
X	XX	Xx
x	Xx	xx

$P(X_{Dominant}) = 3/4$

$P(X_{recessive}) = 1/4$

$P(1D, 2R) = 1/64 + 1/64 + 1/64$

$= 9/64$

X (3/4) Y 1/4 Z 1/4 = 3/64

1/4 (3/4) 1/4 = 3/64

1/4 1/4 (3/4) = 3/64

9/64

b) What is the probability that offspring carries 2 dominant and 1 recessive genes?

P(Dominant) = 3/4
P(Recessive) = 1/4

P(2D,1R) = 9/64 + 9/64 + 9/64
= 27/64

X	Y	Z	
3/4	3/4	1/4	= 9/64
3/4	1/4	3/4	= 9/64
1/4	3/4	3/4	= 9/64

27/64

c) What is the probability that offspring carries 3 recessive genes?

X	Y	Z	
1/4	1/4	1/4	= 1/64 → All recessive
1/4	1/4	1/4	= 1/64
1/4	1/4	1/4	= 1/64

Same, 1/64

P(3R) = 1/64

$C(3,3) = \frac{3!}{0!\,3!} = 1$

1/64